

## **Extracting Growth Cycles from Productivity Indexes**

October 2007

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## 1 Introduction

This paper coincides with the Statistics New Zealand Hot Off The Press release, *Productivity Statistics 1978–2006*. The release adds to the existing understanding of New Zealand's productivity performance in the measured sector<sup>1</sup> by including backdated figures from 1978 to 1987 in addition to the previously released series. The longer time series means it is now more feasible to identify growth cycles in the data.

The analysis of growth cycles has been undertaken to assist users in interpreting the results of the Statistics NZ productivity series, as well as to provide additional insight into the nature of the New Zealand economy over the last three decades. In this context, the benefit of breaking the data into cycles is that this approach better accounts for factors that tend to vary within a cycle, such as capacity utilisation, so that comparisons of productivity performance between periods can be undertaken with the impact of these additional factors minimised.

There are a number of data filters commonly used in practice that aid in identifying business cycles in economic time series. Such filters can also be used to extract growth cycles from productivity data. The application of the filters can differ from the business cycle literature, however, because it is not necessary to assume that business cycles and productivity growth cycles are of the same length.

The filters used in this paper are: the Henderson filter, Hodrick-Prescott filter and Baxter-King filter. The performance of each is compared with known stylised features of the productivity data. Growth cycles, using each of these methods, are calculated for the labour, capital, and multifactor productivity series, as well as the output series. Additionally, supplementary cycles for the labour and capital input series are presented in appendix C.

Due to the specific objective of this exercise, this paper is not intended to provide a comprehensive evaluation of New Zealand's growth cycles, and it should be noted that the models discussed are univariate in nature. Thus, the economic conditions throughout the time period are also considered to provide a more complete analysis. To this end, this paper evaluates the estimated cycles in light of the key economic events throughout the series. A timeline of notable economic events and reforms, as well as supplementary graphs of economic indicators, are presented in appendices A and B. Additionally, when interpreting the results in a wider context, readers should note that the productivity series covers the measured sector, as opposed to the whole economy.

This paper is structured as follows: section 2 provides a brief summary of previous analysis undertaken on New Zealand business cycles. The paper then provides background into the concepts of productivity and the linear filters considered in sections 3 and 4, respectively. Section 5 discusses the results of applying the filters to the multifactor, labour and capital productivity series and output.

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<sup>1</sup>In 2004, the measured sector covered approximately 63 percent of the economy. It excludes the following industries: government administration and defence, health, education, property and business services, and personal and other community services.

## 2 Past research on New Zealand business cycles

A number of papers have been written on New Zealand's business cycles, with the data used and methodologies chosen differing substantially. Papers of note include Haywood (1972), Haywood and Campbell (1976), Kim, Buckle, and Hall (1995) and Hall and McDermott (2007). Due to both the unique objective of this exercise and the new data that is available, the methodology adopted in this paper differs somewhat from previous papers. As noted, the focus of this paper is on the New Zealand productivity series, covering the measured sector, as opposed to the aggregate economy. The univariate nature of the models considered means that consideration of the economic conditions throughout the period is necessary when reviewing the series.

The papers that cover overlapping time periods to this paper include Kim, Buckle and Hall (1995) and Hall and McDermott (2007). The former paper estimates business cycles using a variety of techniques, using quarterly data from 1966 to 1993. The 'benchmark' business cycle turning points derived are derived essentially from the Bry and Boschan method, the results of which are shown to be similar to that of the Hodrick-Prescott, Henderson, and STAMP methods. Peaks in the cycles are estimated as falling within 1982, 1986 and 1989.<sup>2</sup>

Hall and McDermott (2007) estimate business cycles using quarterly output data from 1947 onwards, deriving a set of classical business cycle turning points. In their analysis, they use Markov-switching models, estimated by Gibbs sampling methods to derive mean growth rate and volatility regimes, the properties of which are used to determine cycles. From the analysis, they identify peaks in their series within the years 1982, 1987, 1990 and 1997. They conclude that the diversity in their estimated cycles arise due to disruptions in the pattern of the business cycles, from shocks such as the 1970s oil shocks, and the economic reforms of the mid 1980s to early 1990s.

## 3 Productivity measures

Productivity is an important measure of the efficiency of an economy. It is calculated as the ratio of output to one or more inputs, representing the growth in output not accounted for by the growth of an input or inputs. Statistics NZ's output measure is constant-price GDP<sup>3</sup> and the inputs measured are services from labour and capital inputs. Growth in productivity means that a nation can produce more output from the same amount of input, and is an important contributing factor to a nation's long-term material standard of living.

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<sup>2</sup>The authors note that 1989 peak was estimated "From capita series, omitting the 1992:2 expenditure based outlier".

<sup>3</sup>The output series used in this analysis, and the *Productivity Statistics: 1978–2006* Hot Off The Press release, includes provisional data on measured sector GDP for years prior to 1988. Final GDP data for these years will be introduced to the productivity series in 2008. In future productivity releases, the business cycles presented in this paper will be recalculated, and modified if necessary, with the most updated data available.

### 3.1 Multifactor productivity

Multifactor productivity (MFP) is measured as the ratio of output to combined inputs (labour and capital). MFP, measured residually, represents growth that cannot be attributed to capital or labour, such as technological change or improvements in knowledge, methods and processes.

### 3.2 Labour productivity

Labour productivity is measured as a ratio of output to labour input. Labour input is measured in terms of 'hours paid' in the economy, and includes labour for both wage and salary earners, and self employed workers.

### 3.3 Capital productivity

Capital productivity is measured as a ratio of output to capital input. Capital input is measured in terms of 'capital services', which is measured as the use of capital assets over a specified period of time (annual March years in this case).

## 4 Linear filters

Each of the time series discussed in section 3 can be thought of existing of at least two components, trend and cycle, that are not directly observed. For example

$$y_t = \tau_t + c_t + \epsilon_t$$

where  $\tau_t$  and  $c_t$  are the trend and cycle, respectively, and  $\epsilon_t$  is some irregular component. A linear filter can be applied to the original series to provide a direct estimate of a particular component. For example, the trend may be estimated as

$$\hat{\tau}_t = \sum_{i=S}^T w_i y_{t+i}$$

and the cycle as

$$\hat{c}_t = y_t - \hat{\tau}_t$$

In order to extract the cycle component from each of the productivity measures, a number of linear filters used commonly in such applications were considered. This section contains an overview of three chosen methods, while an expanded discussion of each appears in appendix E. The estimated components are discussed in section 5 below.

A more detailed discussion of linear filters appears in appendix D.

#### 4.1 Hodrick-Prescott filter

The Hodrick-Prescott filter was introduced in a working paper in 1980, but first published in Hodrick and Prescott (1997). The filter was originally intended to help determine business cycles. The model assumes that a time series  $y_t$  is made up of an unobserved, non-stationary time trend,  $\tau_t$ , and a further (weakly) stationary cyclical component  $c_t$ . That is,

$$y_t = \tau_t + c_t, \quad t = 1, 2, \dots, T - 1, T \quad (1)$$

Hodrick and Prescott proposed a method of extracting  $\tau_t$  that requires the minimization of the following cost

$$\text{Min}_{\{\tau_t\}_{t=1}^T} \left\{ \sum_{t=1}^T (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \right\}$$

The first term penalises the variance in the cyclical component, while  $\lambda$  describes how much the lack of smoothness in the trend contributes to the overall cost. Thus, the extracted trend becomes smoother as larger values of  $\lambda$  are chosen.

The Hodrick-Prescott filter is an example of a low-pass filter. In practice, this means that while the trend can be made suitably smooth simply by choosing a sufficiently large  $\lambda$ , any remaining variation is included in the cycle component. The choice of  $\lambda$ , then, is a somewhat contentious issue. For the annual data presented here, a value of 6.25 was used. Appendix E includes analysis for the choice of this parameter.

The Hodrick-Prescott filter is easy to apply in practice, and features heavily in the business cycle literature.

## 4.2 Baxter-King filter

The Baxter-King filter is an example of a band-pass filter. An ideal band-pass filter is one that removes all components that do not correspond to a suitably chosen time interval. For example, Baxter and King (1999) proposed that the typical length of US business cycles lay between 1.5 and 8 years. That is, cycles of longer than 8 years should be suppressed, as should those shorter than 1.5 years.

Of course, the ideal filter requires an infinite time series. In this case, an alternative finite approximation of order  $2k + 1$  is used:

$$\hat{c}_t = \sum_{j=-k}^k \alpha_j y_{t-j}$$

where  $k$  is selected as appropriate. This provides a direct estimate of the cycle itself, and  $y_t - \hat{c}_t$  the trend. It should be noted that the resulting trend is likely to be less smooth than approximate high-pass filters such as the Hodrick-Prescott filter. This is because removing the cycle in the case of the Baxter-King filter leaves both high and low frequency components, while applying a high-pass filter leaves only high frequency signals.

As for other filters, the choice of parameters can be somewhat subjective. Regardless, the practice of fixing cycles between 1.5–2 and 8 years is common. The filter length is also often set to be three times the frequency of the time series under consideration.

## 4.3 Henderson filter

The Henderson filter was originally derived in 1916 by Henderson for actuarial purposes. However, the filters are now commonly used to smooth time series data and, in particular, are utilised within the X11 and X12 seasonal adjustment programs to estimate trend components of seasonal series.

The Henderson filter can be described as a special case of linear polynomial regression. Suppose there is a time series  $y_t$  decomposable as follows:

$$y_t = \tau(t) + c_t$$

We assume  $E(c_t) = 0$ , and that  $\tau(t)$  is a deterministic function in time such that  $E(y_t) = \tau(t)$ . If  $\tau(t)$  is differentiable, then it can be approximated locally by a polynomial,  $v(t)$ . That is,

$$\begin{aligned}\tau(t+j) &\approx v(t+j) \\ &= \sum_{k=0}^p \beta_k j^k\end{aligned}$$

The Henderson filter arises where  $p = 3$ , and the weights are chosen in such a way that  $\text{var}[\Delta^3 \hat{\tau}(t)]$  is minimised, subject to the constraints<sup>4</sup>

$$\sum_{j=-h}^h w_j = 1 \text{ and } \sum_{j=-h}^h w_j j^k = 0, \quad k = 1, 2, 3$$

In the case of the Henderson filter, a well known set of asymmetric weights exist that can be used at the end-points where there are missing observations. This is known as the Musgrave method, and the weights are collectively referred to as a surrogate Henderson filter (see appendix E).

Thus, the Henderson filter is similar to the Hodrick-Prescott filter – the filtered series will be smooth, and the cycle can be extracted by detrending the original series. However, the estimated trend produced by applying the Hodrick-Prescott filter will generally be straighter than the Henderson. This should not be entirely surprising, since the Henderson filter is designed to follow a cubic polynomial locally.

## 5 Results

This section provides a summary of the cycles derived for labour, capital and multifactor productivity, as well as output for the measured sector. Cycles are defined using a peak-to-peak definition, meaning that cycles are considered to commence at points where the deviation above the trend is highest, as illustrated in the graphs that follow. The rationale for this is that for consistent comparisons it is ideal to compare productivity between periods of similar capacity utilisation, and the peaks are assumed to be periods of high capacity utilisation. To supplement this analysis, a timeline of key economic events and reforms are provided in appendix A. A number of these events are highlighted in the commentary below. One recurring theme of the analysis is that the shocks to the economy, and the large number of reforms that occurred throughout the 1980s and early 1990s have influenced the length of the cycles for this period. In addition to the chronology of events, appendix B includes graphs of economic indicators including the inflation rate, the unemployment rate, the terms of trade, the Trade Weighted Index (TWI) and NZ/USD exchange rate, as well as 90-day bill and 10-year bond yields, all of which provide some explanation to the growth in economic activity over the period.

<sup>4</sup>  $\Delta$  is the forward difference operator. That is,  $\Delta y_t = y_{t+1} - y_t$ .

More detailed information about the average changes in the productivity series within the cycles are available in the Hot Off The Press release *Productivity Statistics: 1978–2006*. In presenting the results of the series, the cycles estimated from the MFP series below have been used, as opposed to using separate cycles for each productivity series. Using a single set of cycles in analysis allows easier comparisons of the behaviour of the series over the period. MFP has been chosen due to its importance in productivity measurement and its property of being a function of labour input, capital input and output, all of the series that are used to compile the productivity series. It is also the series chosen in recent literature, such as the Australian Bureau of Statistics (2007).

Table 5.1

**Growth Cycle Starting Points**

Year	Multifactor Productivity	Labour Productivity	Capital Productivity	Output
1980				HPB
1981				
1982	HPB	HPB	HPB	HPB
1983				
1984		PB		
1985	HPB	H	HPB	HPB
1986				
1987	HPB	H	HPB	HPB
1988				
1989			H	
1990	HPB	HPB		
1991				H
1992				
1993				
1994	HB	HPB		
1995	P		HPB	
1996				
1997	HPB		HPB	HPB
1998		HPB		
1999				
2000	HPB	HPB	HPB	HPB
2001				
2002				
2003	HPB	HPB	HPB	HPB

**Note:** H = Henderson filter  
P = Hodrick-Prescott filter  
B = Baxter-King filter

Table 5.1 summarises the likely starting points of growth cycles for each of the three methods. As the summary above shows, the filters generally match well for for each of the series. As noted, the cycles estimated for MFP have been chosen, due to MFP incorporating the data from the labour, capital and output series. The Hodrick-Prescott filter is the preferred method for determining the growth cycles and, as such, is used as the benchmark for determining starting points in the *Productivity Statistics:*

1978–2006 release. Further details of each filter is contained in Appendix E. Additionally, due to the fact that the latest years' cycles are hard to determine, cycles for the most recent years have not been considered. Similarly, observations at the beginning of a series should be treated with some caution. A more detailed discussion of the end-point problem appears in appendix D.

For the final series, based on MFP results, cycles commencing from 1978, 1982, 1985, 1990, 1997 and 2000 have been selected. The reasoning for selecting the chosen cycles is explained in the following subsection. Adding these cycles to the analysis of the series provides the following annual average movements. These are discussed in detail throughout the Hot Off the Press release, *Productivity Statistics: 1978–2006*.

Table 5.2 summarises the average growth in the series, accompanied by changes in some key economic indicators over the period.

Table 5.2

**Summary of key economic indicators**

Average annual growth rates							Average Rate	
Year ended March								
Cycle	Output	Labour productivity	Capital productivity	Multi-factor productivity	CPI	Terms of Trade	Unemployment <sup>(1)</sup>	TWI <sup>(2)</sup>
	Percent							
1978–1982	2.1	1.7	0.4	1.2	14.6	-0.3		96.5
1982–1985	3.4	1.4	-3.1	-0.4	9.6	-1.5		74.8
1985–1990	0.7	2.9	-4.0	0.2	10.9	4.4	5.4	62.6
1990–1997	3.2	2.6	1.3	2.1	2.5	-1.1	8.3	58.9
1997–2000	2.6	3.5	0.6	2.3	0.8	-0.5	6.9	59.3
2000–2006	3.4	1.4	-0.1	0.7	2.6	1.9	4.7	59.6
1978–2006	2.6	2.2	-0.6	1.1	6.2	0.6		

(1) Average unemployment rates calculated from March 1986. Prior data is unavailable.

(2) Average TWI rate for the 1978-1982 cycle calculated from January 1981. Prior data is unavailable.

## 5.1 Multifactor productivity

MFP cycles commence at 1978, 1982, 1985, 1987, 1990, 1994–1995, 1997 and 2000 (see figures 5.1 and 5.2). Being a function of labour input, capital input and output, MFP is driven by the movements in all of the three series, so the similarity of the estimated cycles to those of the other series is expected. The peaks in the movements from the trend in 1987 and 1995 are debatable, given that they are very close to the peaks in 1985 and 1997 respectively, and appear to be upward movements within cycles, rather than peaks in the series. Although there were events in the economy that caused a downturn around 1987, the deviation from the trend growth rate in MFP was minimal, as shown in figure 5.2. The movement from the trend was more significant in 1995, however the economic events around 1997, including the Asian currency crisis and the severe droughts around this period provide more rationale for the turning point to be around 1997.

The peak in 1990 is not the most pronounced, but the significant economic events and reforms that occurred around this period provide some economic rationale for a cycle to begin around this time. Discussion of the movements in the components of MFP is presented in the following subsections.

Figure 5.1

**Trend Component for Multifactor Productivity**  
*Year ended March*  
Base: 1996 (=1000)

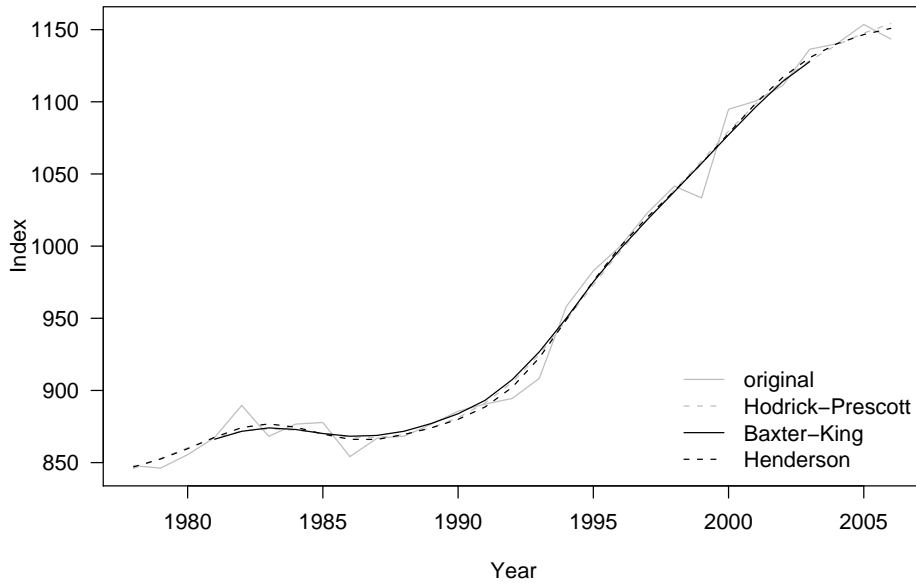
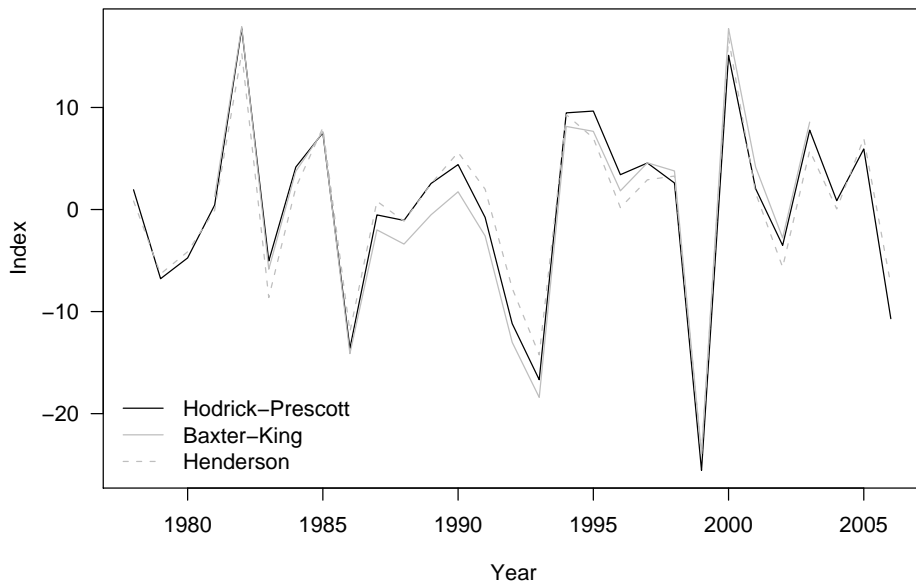


Figure 5.2

**Cycle Component for Multifactor Productivity**  
*Year ended March*



## 5.2 Labour productivity

Depending on which filter is chosen, growth cycles for labour productivity begin in 1978, 1982, 1984–1985, 1987, 1990, 1994, 1998 and 2000 (see figures 5.3 and 5.4). Again, the filters provide relatively consistent cycles, with a clear peak in 1990. For the total economy, the average unemployment rate rose from 5.4 percent in 1985–1990 to 8.3 percent in 1990–1997, with unemployment reaching record levels in 1991. The increase in unemployment between these periods was most evident in the measured sector. This record high in measured unemployment was not sustained, and, despite significant rises in 1998, the trend unemployment rate has been falling since this period, with unemployment reaching record lows of 3.6 percent in later quarters of the series.

The low employment over the late 1980s and early 1990s was one contributor to the relatively high labour productivity growth, which averaged 2.9 and 2.6 percent annually in 1985–1990 and 1990–1997, respectively. One theory about why this is often the case in periods of low employment is that the skill composition of the employed labour force changes, due to the more skilled providing a higher proportion of labour. Due to the relationship between labour and capital in production, the large investment in capital over the 1980s could be another factor driving the rise in labour productivity over this period. Reforms over the period that affected the labour market included the introduction of the Employment Contracts Act in 1991 and its successor, the Employment Relations Act in 2000.

As for the MFP series, there is some evidence of a peak appearing in 1987, but there is also some indication of a peak in 1998. The peak in 1987 is estimated from the Henderson filter, but this is not supported by the other filters. The movements from the trend around 1998 are expected, given the shocks to the economy around this period. However, the movement from the trend is still marginal.

Being a partial measure of productivity, labour input, and thus labour productivity, is affected by movements in capital input, and vice-versa, so the movements in the capital series should also be considered when interpreting this series.

Figure 5.3

**Trend Component for Labour Productivity**  
*Year ended March*  
Base: 1996 (=1000)

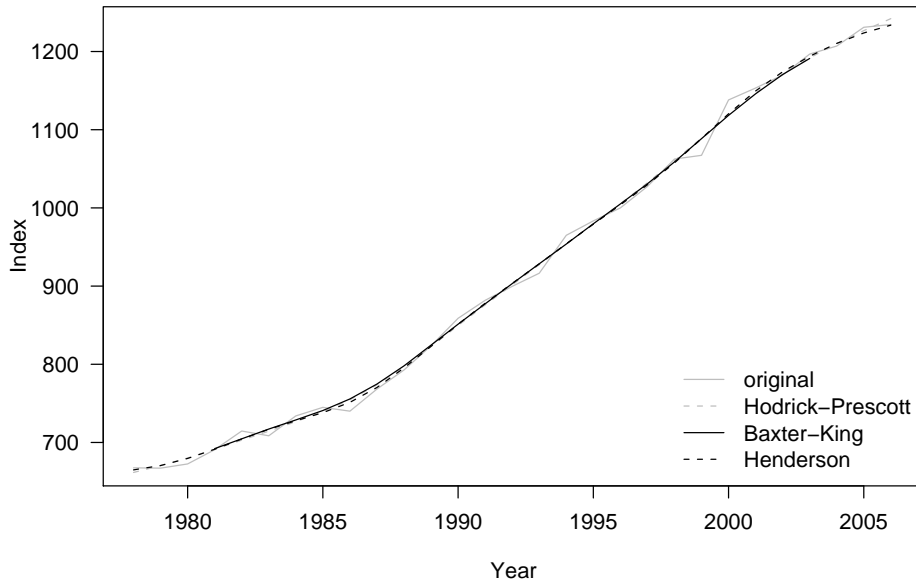
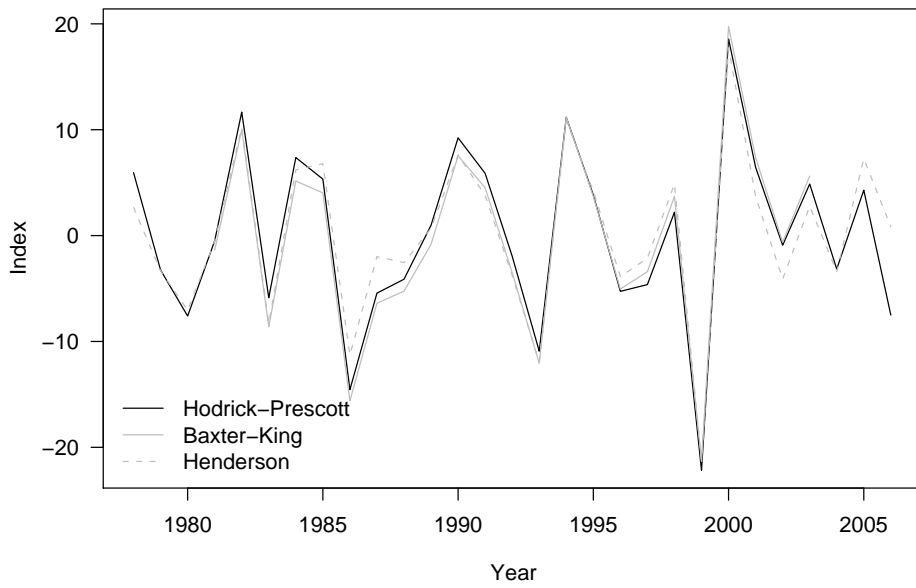


Figure 5.4

**Cycle Component for Labour Productivity**  
*Year ended March*



### 5.3 Capital productivity

Estimated capital growth cycles commence from 1978, 1982, 1985, 1987, 1989, 1995, 1997 and 2000 (see figures 5.5 and 5.6). The most significant difference between the capital productivity growth cycles and the others in question are that it is only the Henderson that indicates a cycle beginning in 1989, with no suggestion of a peak at 1990.

The most notable series of capital investments within this timeframe were the 'Think Big' projects in which large-scale infrastructure investments, largely aimed at increasing local energy production, commenced progressively through the early 1980s. This series of construction projects included the methanol plant at Waitara, ammonia/urea plant at Kapuni, synthetic petrol plant at Motunui, expansion of the oil refinery plant at Marsden Point, expansion of the New Zealand steel plant at Glenbrook, electrification of the North Island main trunk railway between Te Rapa and Palmerston North, a third reduction line at the Tiwai Point aluminium smelter and Clyde Dam at the Clutha river. The high capital investment combined with the relatively low output growth in the early 1980s was a key driver to the negative capital productivity growth that was exhibited in this period. However, capital deepening can promote productivity growth in later periods.

In addition to the economic events previously mentioned, other factors such as the lower nominal interest rates and inflation from the early nineties also influenced capital investment. Being a partial measure of productivity, capital productivity is affected by movements in the labour series so the movements in the capital input should also be considered when interpreting this series.

Figure 5.5

**Trend Component for Capital Productivity**  
Year ended March  
Base: 1996 (=1000)

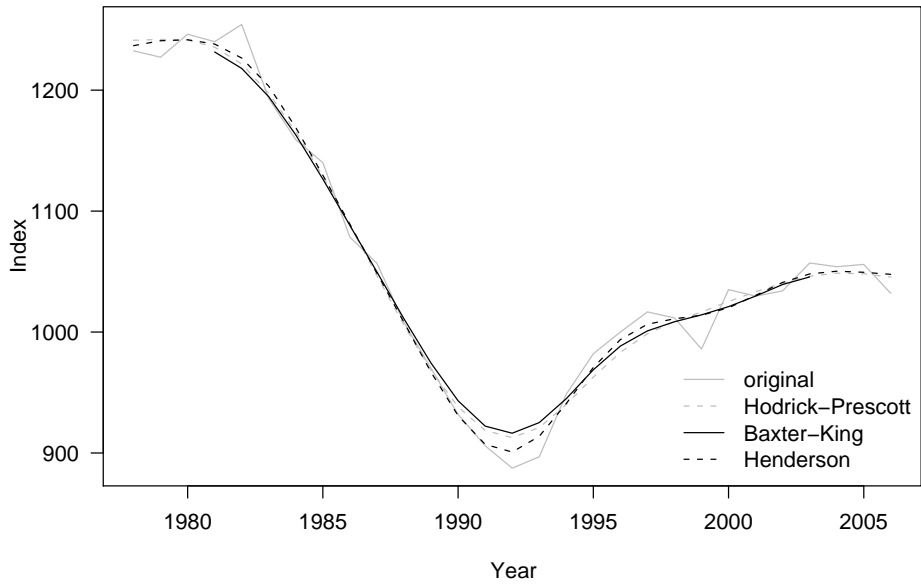
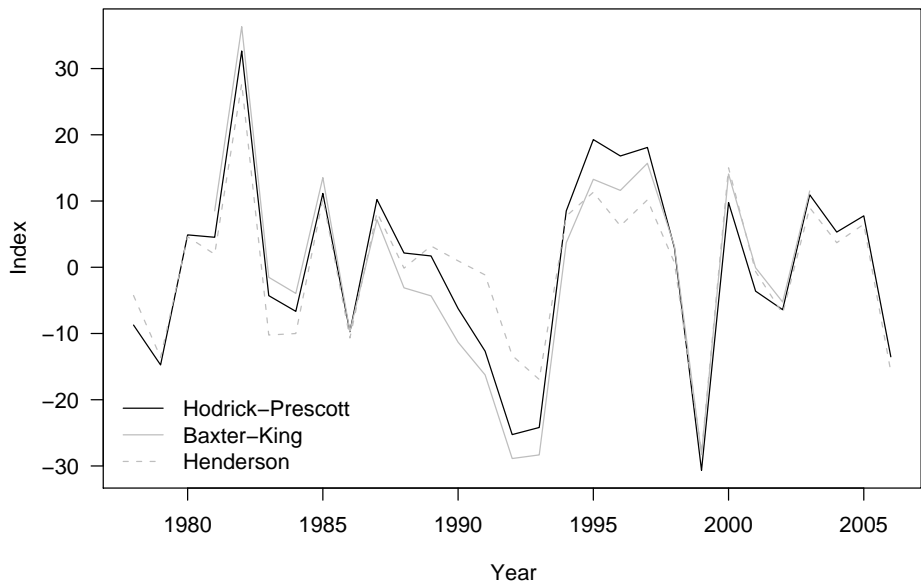


Figure 5.6

**Cycle Component for Capital Productivity**  
Year ended March



## 5.4 Output

The cycles for output are relatively consistent for the three filters, aside from the cycle beginning in 1991, as only the Henderson filter indicates this (see figures 5.7 and 5.8). Despite this, the significant changes that were exhibited in this year, and the likely lagged effects of economic events and reforms of the late 1980s, provide some economic rationale for the change. In the late 1970s, New Zealand's economy was adversely affected by a number of well documented factors, including the inflationary pressure led by the oil price shocks. Despite a price freeze in 1982, inflation remained high until the Reserve Bank Act of 1989, which resulted in inflation targeting within the band of 0 to 2 percent, which was later revised to 0 to 3 percent and to 1 to 3 percent in 1996 and 2002, respectively. International events, such as the Asian economic crisis of 1997, clearly had a large impact on the NZ economy, reflected by the break in the relatively strong economic growth that was exhibited from 1994. Severe droughts around this time also affected the agricultural sector significantly. From 1999, the economy returned to a relatively steady growth path. However, output growth has slowed in recent years, which in turn has led to lower than average MFP growth.

Figure 5.7

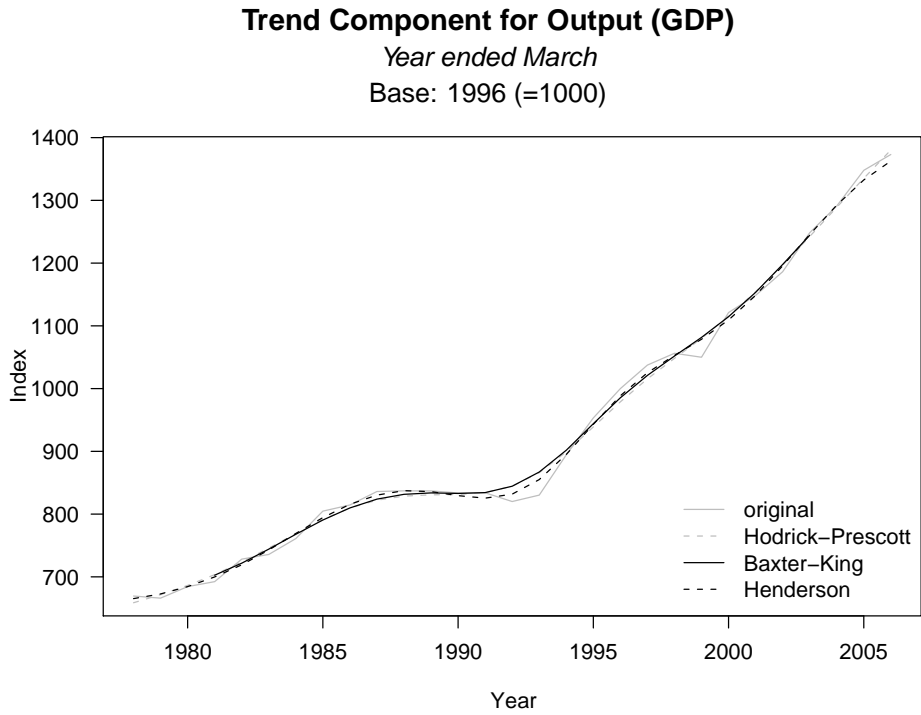
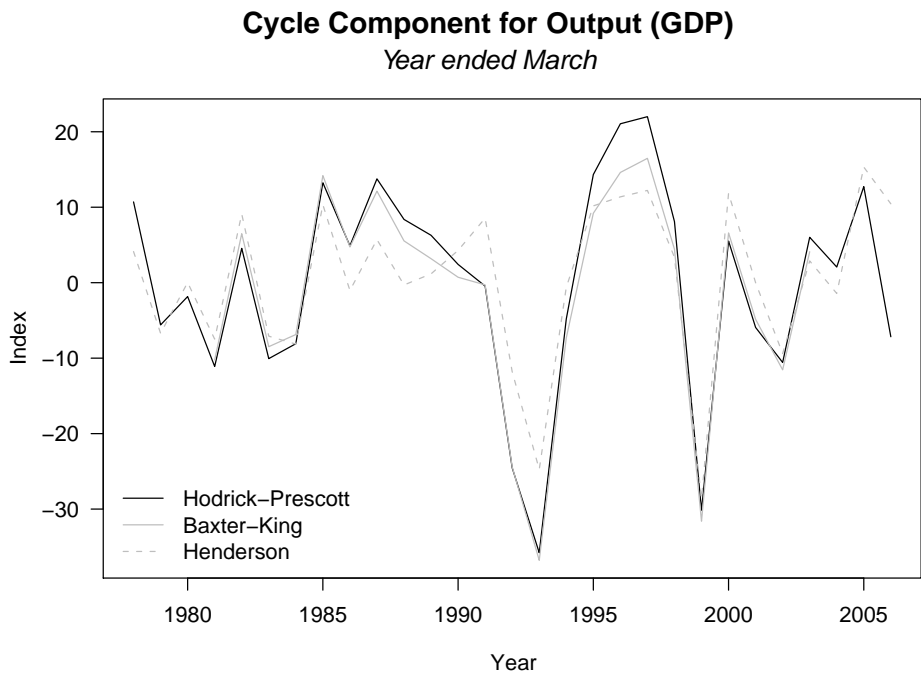


Figure 5.8



## 6 Conclusion

The main purpose of this exercise has been to identify growth cycles to assist users in interpreting the results of the Statistics NZ productivity series. As noted, year-on-year comparisons of productivity movements can be problematic, due to capacity utilisation of capital varying within cycles. The final set of growth cycles was estimated using the Hodrick-Prescott filter, with the results of other filters and the economic events throughout the period being considered as well. The growth cycles estimated under the methods used are generally very consistent, although it is noted that these are just some of the approaches available for determining growth cycles. The resulting estimated growth cycles have clearly been affected by economic shocks and reforms, leading to cycles of varying length.

Growth cycle analysis will be included in upcoming Statistics NZ productivity releases, with updates to be added as the time series expands and as ongoing improvements to the series are incorporated.

## A Chronology of major economic events and reforms 1978-2005

### 1978

November 1978 General Election held.

### 1979

A number of export-oriented subsidies introduced/ raised, including the minimum payments for some agricultural production and export subsidies being introduced for non-traditional exports, such as manufactured goods.

June The government moves to a 'crawling peg' system to control the exchange rate, in which the Reserve Bank were permitted to devalue the dollar by up to 0.5 percent per month without involving the Minister of Finance. This ran until 1982.

The government announces the 'Think Big' strategy, incorporating a number of large-scale construction projects over the following years.

June More oil price shocks spark further inflationary pressure.

### 1981

November 1981 General Election held.

### 1982

June The Government announces a freeze on prices in the domestic economy, covering wages and salaries, interest rates and exchange rates. This ran until February 1984.

### 1983

January New Zealand and Australia sign the Closer Economic Relations (CER) trade agreement.

### 1984

February Price freeze, imposed in June 1982, lifted. Wage freeze extended until an agreement on long-term wage fixing procedure is reached.

June Supplementary Minimum Payments (SMPs) to farmers removed from the end of the 1983/84 season.

July Price freeze re-introduced for 3 months.

1984 General Election held.

NZ dollar devalued 20 percent.

The Reserve Bank announces the closure of the foreign exchange market due to its reserves of overseas currencies being close to exhausted.

July/August Controls on interest rates and credit growth abolished.

October New wage-fixing rules introduce greater flexibility in bargaining and wage freeze, imposed in June 1982, lifted.

November Budget announces intention to eliminate numerous consumer and producer subsidies/incentives.

	Price freeze, re-introduced in July, lifted.
December	All controls on both outward and inward foreign exchange transactions removed. Controls on overseas borrowing removed. New motor vehicle industry plan allows for greater access for imported vehicles and components.
<b>1985</b>	
February	Compulsory ratio system requiring financial institutions to invest fixed proportions of their total funds in government and public securities abolished.
March	New Zealand dollar floated.
<b>1986</b>	
1986	Complete phasing-out of permits for long-distance road haulage. Progressive removal of imposts on road transport, including excise taxes on fuel and reduced import tariffs on tyres and trucks.
July	Government assumes responsibility for major project and producer board debts totalling \$7.2 billion.
October	Goods and Services Tax (GST) of 10 percent introduced. Compensating reductions in personal tax and benefit increases accompany the new tax. Some sales duties/taxes also removed or reduced.
<b>1987</b>	
	Ten new state corporations (SOEs) established.
June	Government announces intention to sell assets to pay off public debt.
August	1987 General Election held.
August	Labour Relations Act 1987. Rationalises existing legislation under one statute and reforms laws governing labour organisations to improve negotiating environment.
October	World sharemarket crash. New Zealand sharemarket suffers its biggest ever one-day fall.
<b>1988</b>	
1988	Remaining price controls on petrol abolished.
March	Sale of New Zealand Steel to Equiticorp begins the Government's programme of state-owned asset sales.
April	State Sector Act 1988 comes into force. The Act restructures public sector management, introducing increased management flexibility for senior public servants along with increased accountability for performance, and aligns public sector with private sector employment regulation.
July	Import licensing ends for most goods.

Extracting Growth Cycles from Productivity Indexes

August Australian and New Zealand Prime Ministers sign an agreement to bring forward the date for establishing a trans-Tasman free market to July 1990 under the terms of the Australia-New Zealand Closer Economic Relations Trading Agreement. Protocols signed to remove most trade barriers between Australia and New Zealand.

**1989**

July GST increased to 12.5 percent.

December Reserve Bank of New Zealand Act passed. As from February 1990 the Act defines the objective of monetary policy to be the achievement and maintenance of stability in the general level of prices.

**1990**

March First Reserve Bank policy targets agreement signed. The Minister of Finance and the Governor of the Reserve Bank reaffirm the objective of price stability and set an annual CPI increase of 0–2 percent by December 1992 as a target.

October 1990 General Election held.

December New Reserve Bank target agreement signed extending the achievement of the price stability target to December 1993.

Economic package introduces across-the-board benefit cuts from April 1991.

Compulsory union membership ends.

**1991**

May Employment Contracts Act introduced.

**1992**

July ACC reforms introduced. The scheme's source of funds are realigned reducing the share incurred by business and increasing personal contributions. Entitlement redefined, including the abolition of lump sum payments.

December The Government and Reserve Bank sign a new policy targets agreement requiring the Reserve Bank to keep 12-monthly increases in the CPI within the range of 0–2 percent.

**1993**

November 1993 General Election held.

December Agreement reached in the Uruguay round of the General Agreement on Tariffs and Trade (GATT). The agreement will lead to a more open world trading environment through the agreed process of progressive liberalisation.

**1994**

The Government passes the Fiscal Responsibility Act, setting out principles for fiscal management.

**1995**

November The Royal assent to the \$170 million Tainui Māori land settlement in compensation for lands confiscated in 1884 is signed.

**1996**

- July Income tax rates for individual taxpayers reduced.
- September The Government records its third successive financial surplus. The surplus for the 1995/96 financial year was \$3,314 million following \$2,695 million in 1994/95.  
A new policy targets agreement is signed, broadening the permitted 12-monthly increases in the CPI to a range of 0–3 percent.
- October 1996 General Election held.

**1997**

- 1997 The Asian currency crisis begins

**1998**

- 1998 Eastern districts of New Zealand experience severe drought through spring, summer and autumn of 1997/98, seriously affecting agricultural production. Government announces rural sector assistance programme.
- February The Auckland power crisis begins, with full power not returning for 5 weeks.
- June The European Central Bank (ECB) is established in Germany.
- July Income tax rates for individual taxpayers reduced.
- September International credit rating agency Moody's downgrades the credit rating of New Zealand's long-term sovereign debt from AA1 to AA2.

**1999**

- 1999 Widespread contingency planning begins in expectation of the Year 2000 (Y2K).
- February The 90-day bank bill rate goes below 4 percent, an all-time low.
- September New Zealand signs a free-trade accord with Chile.
- November 1999 General Election held.

**2000**

- April The personal income tax rate is increased for those earning over \$60,000 from 33 cents to 39 cents in the dollar.  
The New Zealand Cabinet agrees on a five-year freeze on unilateral tariff reduction.
- October The Employment Relations Act comes into force. The aim of the act is to provide balance in the conduct of employment relationships and enable good faith bargaining. It makes provisions both for employees who join a union and those who do not.

**2001**

- January The Closer Economic Partnership (CEP) between New Zealand and Singapore comes into force. The agreement removes restrictions on trade in goods and services, and on investment, between New Zealand and Singapore.

- May In the 2001 Budget, the Government announces plans for increased funding for participation and quality of early childhood education and the introduction of the National Certificate of Educational Achievement (NCEA).
- July The 'community wage' reverts back to a work-tested unemployment benefit and a non-work tested sickness benefit.
- September The events to become widely known as 'September 11' occur in the United States.  
Parliament passes the Dairy Industry Restructuring Act allowing the New Zealand Dairy Board, the last major producer board, to be abolished and replaced by the Fonterra Co-op.

## 2002

- February The Government releases its policy framework for economic transformation, 'Growing an Innovative New Zealand'. The framework has three key elements:
- strengthening the economic foundations
  - investment in innovation, talent and global connectedness
  - sectoral policies focussing on the bio-technology, ICT and creative sectors.

July 2002 General Election held.

- September New Zealand and Australia sign a Closer Economic Partnership agreement with South-East Asian countries, which plans to cut trade barriers between the two regions with the aim of doubling trade and investment by the year 2010.  
A new policy target agreement is signed. The most significant change is the requirement that the Reserve Bank keep inflation within a 1 to 3 per cent range "on average over the medium term", thereby giving the Reserve Bank more flexibility to respond to shocks in the economy.

## 2003

- The Severe Acute Respiratory Syndrome (SARS) epidemic emerges in a number of countries.
- September The Government announces its tariff policy for post-2005. The highest tariff rates of between 17 to 19 percent will be reduced to 10 per cent by 1 July 2009. Tariff rates on all other goods will be reduced to 5 percent by Jul 2008. Alternative specific tariffs will revert to the apparel ad valorem tariffs on 1 July 2005.

## 2004

- May In the 2004 Budget, the Government announces the new working for families welfare scheme.
- October Oil prices reach record highs, with the West Texas Intermediate price rising to over US\$54 a barrel.
- December New Zealand and Thailand conclude a Closer Economic Partnership (CEP). The CEP involves preferential liberalisation of trade in goods investment and the scope for cooperation in areas such as standards and conformance, competition policy, labour and environment issues and technology transfer.

**2005**

- May Minister for the Environment Pete Hodgson announces details of the proposed Kyoto Protocol based carbon tax. It will commence in April 2007, and be set at \$15 per tonne of carbon dioxide. This will apply to 2012.
- August The price of Brent crude oil hits a record high of US\$70.58 a barrel following Hurricane Katrina. The storm hits the southern coast of the United States causing loss of life, widespread destruction to oil drilling and refining facilities, and flooding to the city of New Orleans and surrounding areas.
- September 2005 General Election held.

Sources: Dalziel and Lattimore (2001), Reserve Bank of New Zealand (2007), Statistics NZ.

## B Graphs of New Zealand economic indicators

Figure B.1

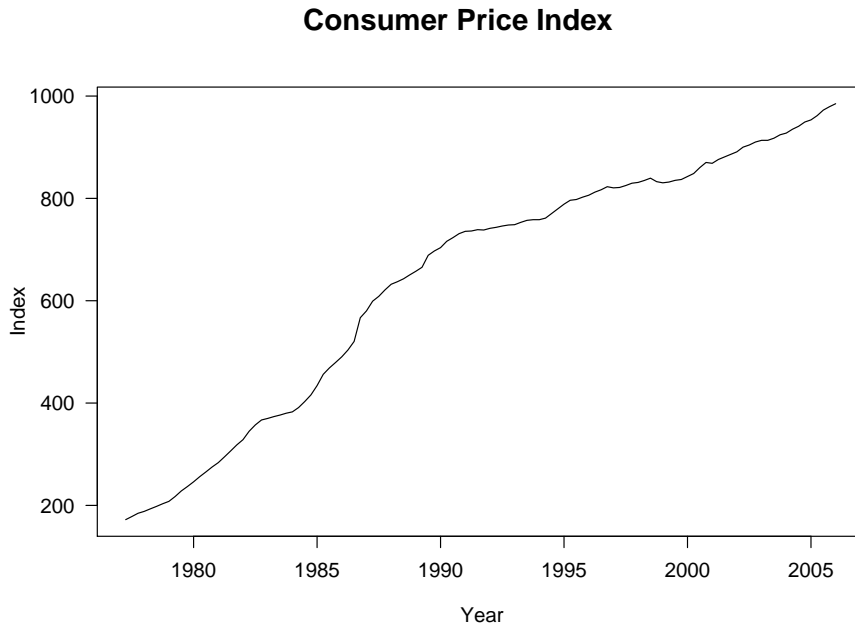
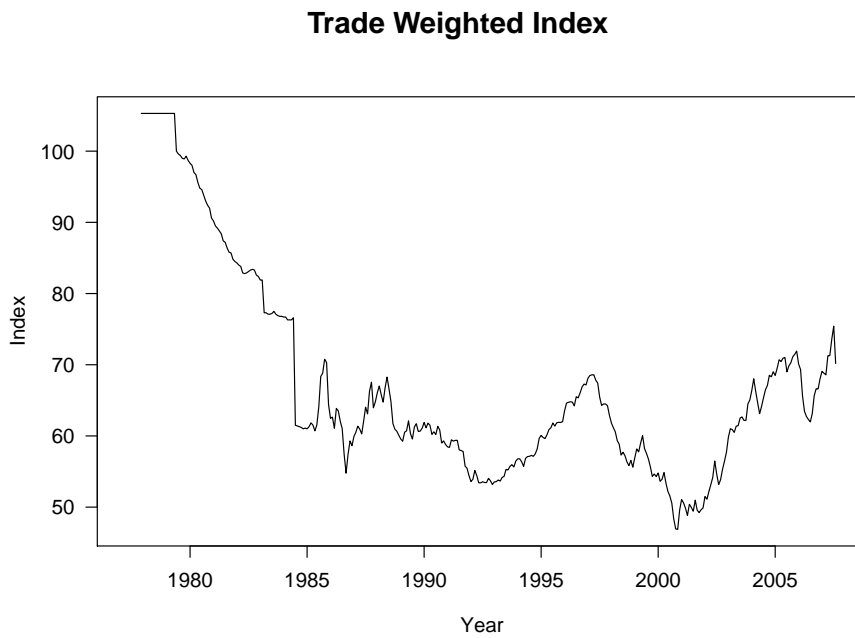


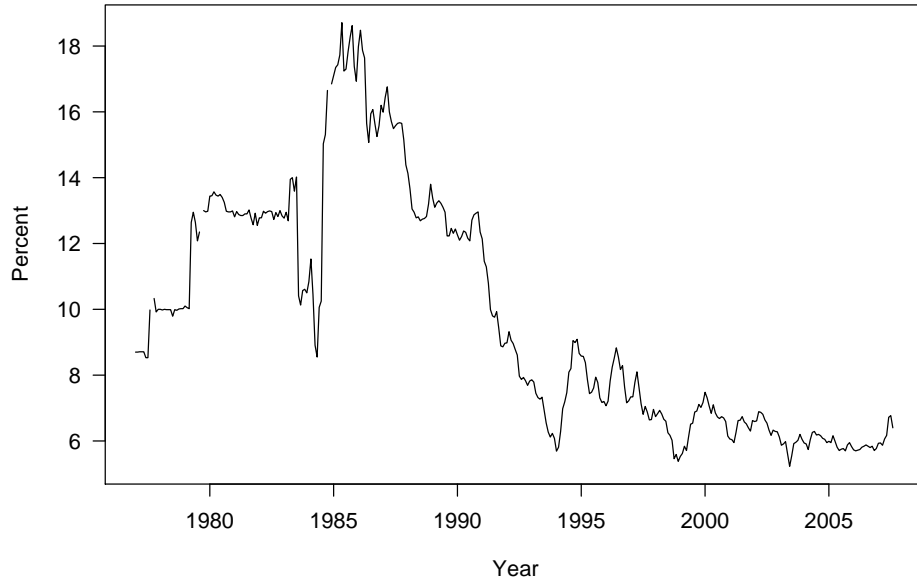
Figure B.2



Source: Reserve Bank of New Zealand.

Figure B.3

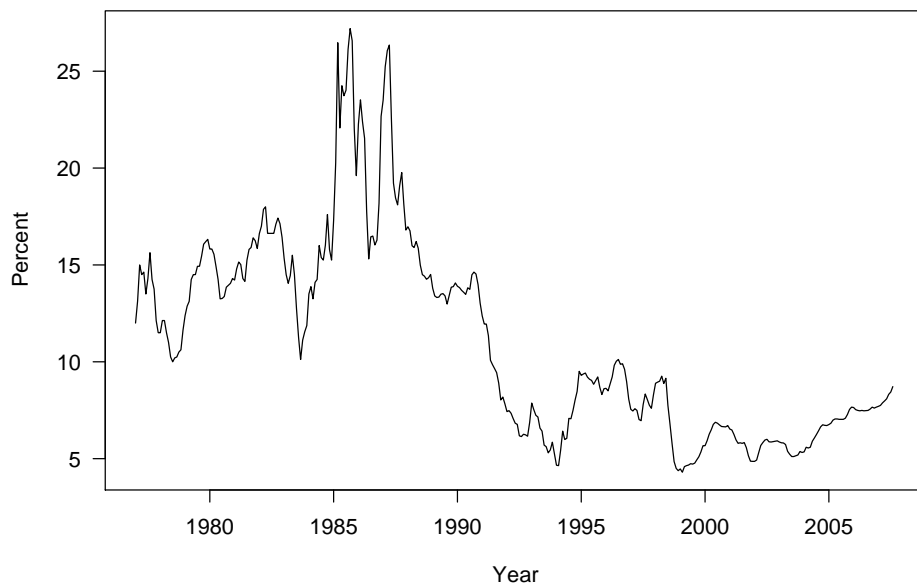
### 10-Year Government Bond Yield



Source: Reserve Bank of New Zealand.

Figure B.4

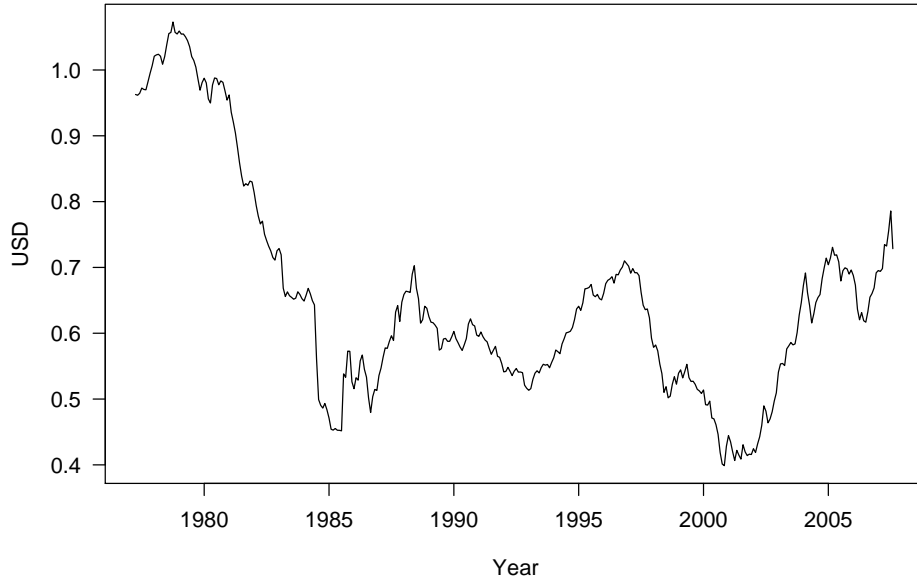
### 3-Month Bank Bill Yield



Source: Reserve Bank of New Zealand.

Figure B.5

### NZD/USD Exchange Rate



Source: Reserve Bank of New Zealand.

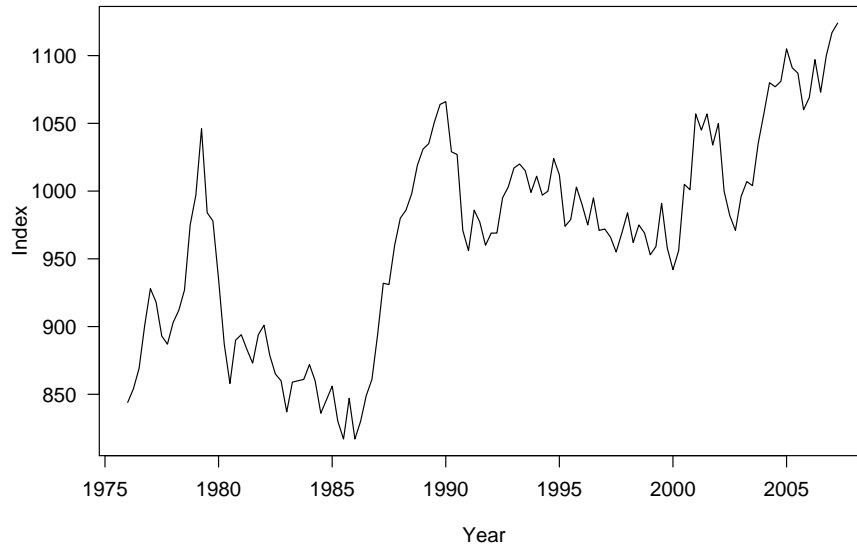
Figure B.6

### Unemployment Rate



Figure B.7

**Terms of Trade**



## C Supplementary cycles: Capital and labour input

Figure C.1

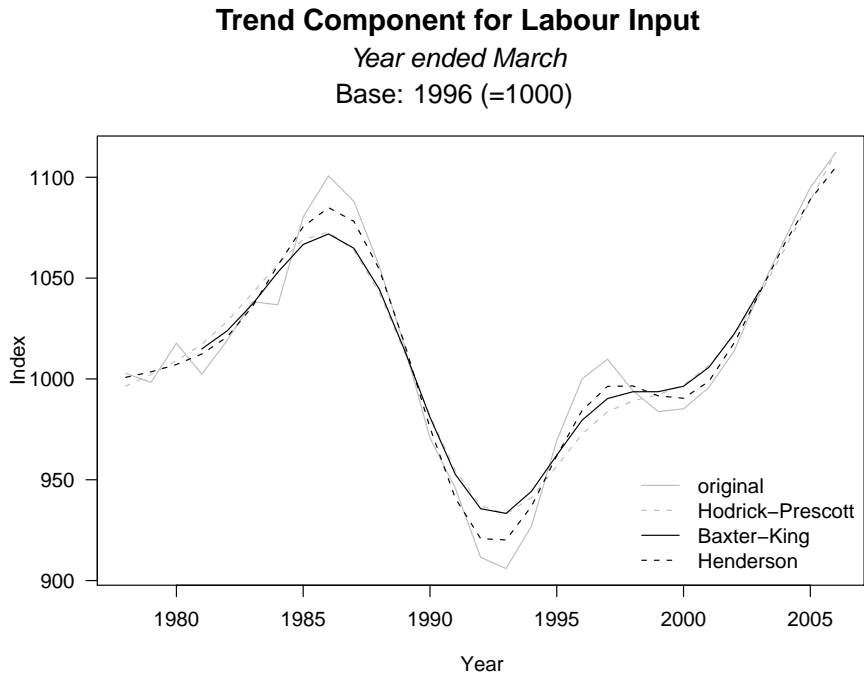


Figure C.2

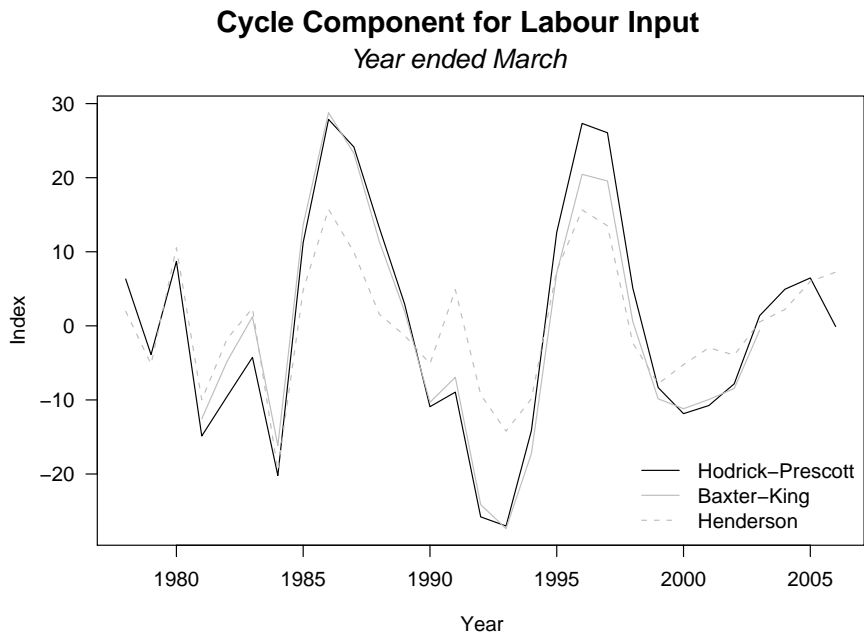


Figure C.3

**Trend Component for Capital Input**  
*Year ended March*  
Base: 1996 (=1000)

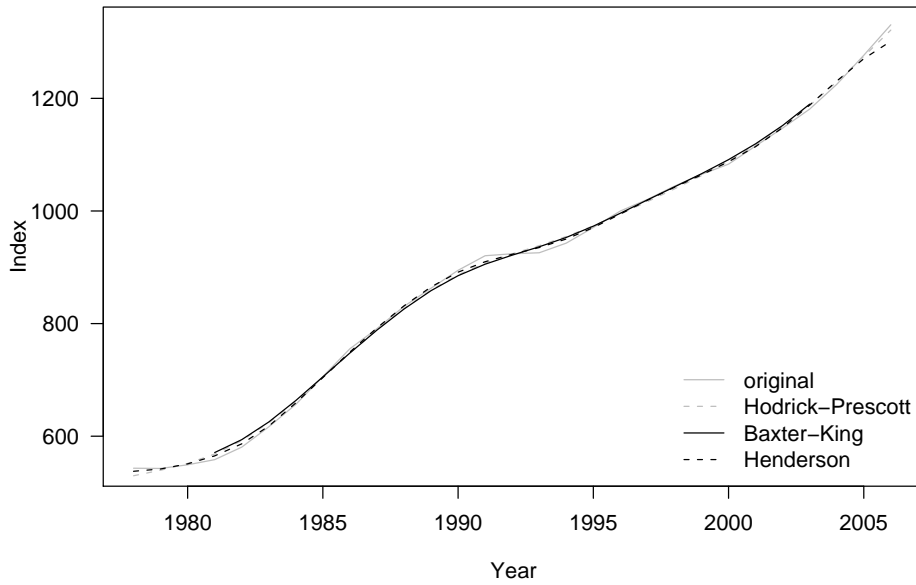
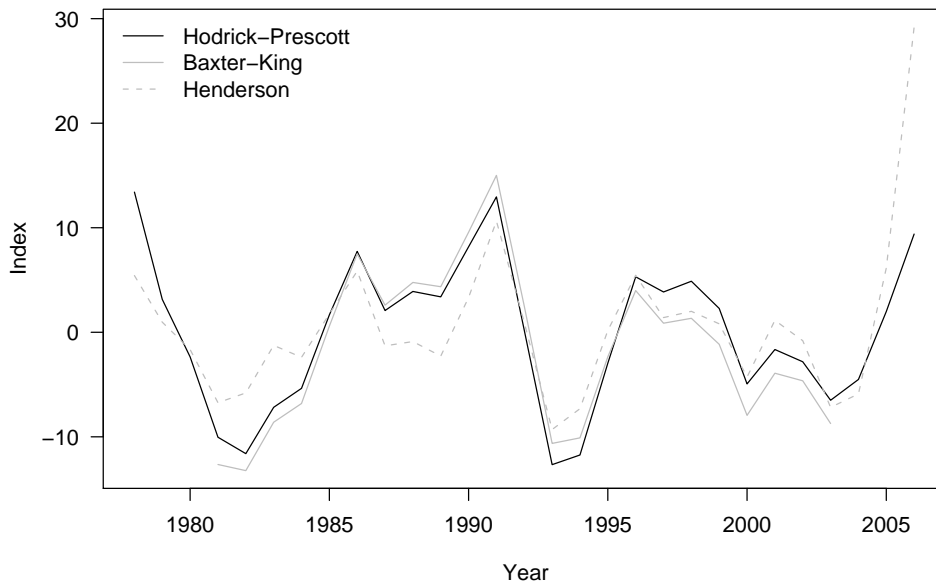


Figure C.4

**Cycle Component for Capital Input**  
*Year ended March*



## D Linear filters and the end-point problem

Let  $y_t$  be a time series, and  $\tilde{y}$  be the series

$$\tilde{y} = \sum_{i=S}^T w_i y_{t+i}$$

where  $\sum_{i=S}^T w_j^2 < \infty$ . This operation is called a linear filter. Further, let the same time series  $y_t$  consist of unobserved components  $\tau_t$  and  $c_t$ , referred to loosely as trend and cycle, respectively. That is

$$y_t = \tau_t + c_t$$

A linear filter can be applied to  $y_t$  in order to estimate the unobserved components. For example, let

$$\hat{\tau}_t = \sum_{i=S}^T w_i y_{t+i}$$

and, then,

$$\hat{c}_t = y_t - \hat{\tau}_t$$

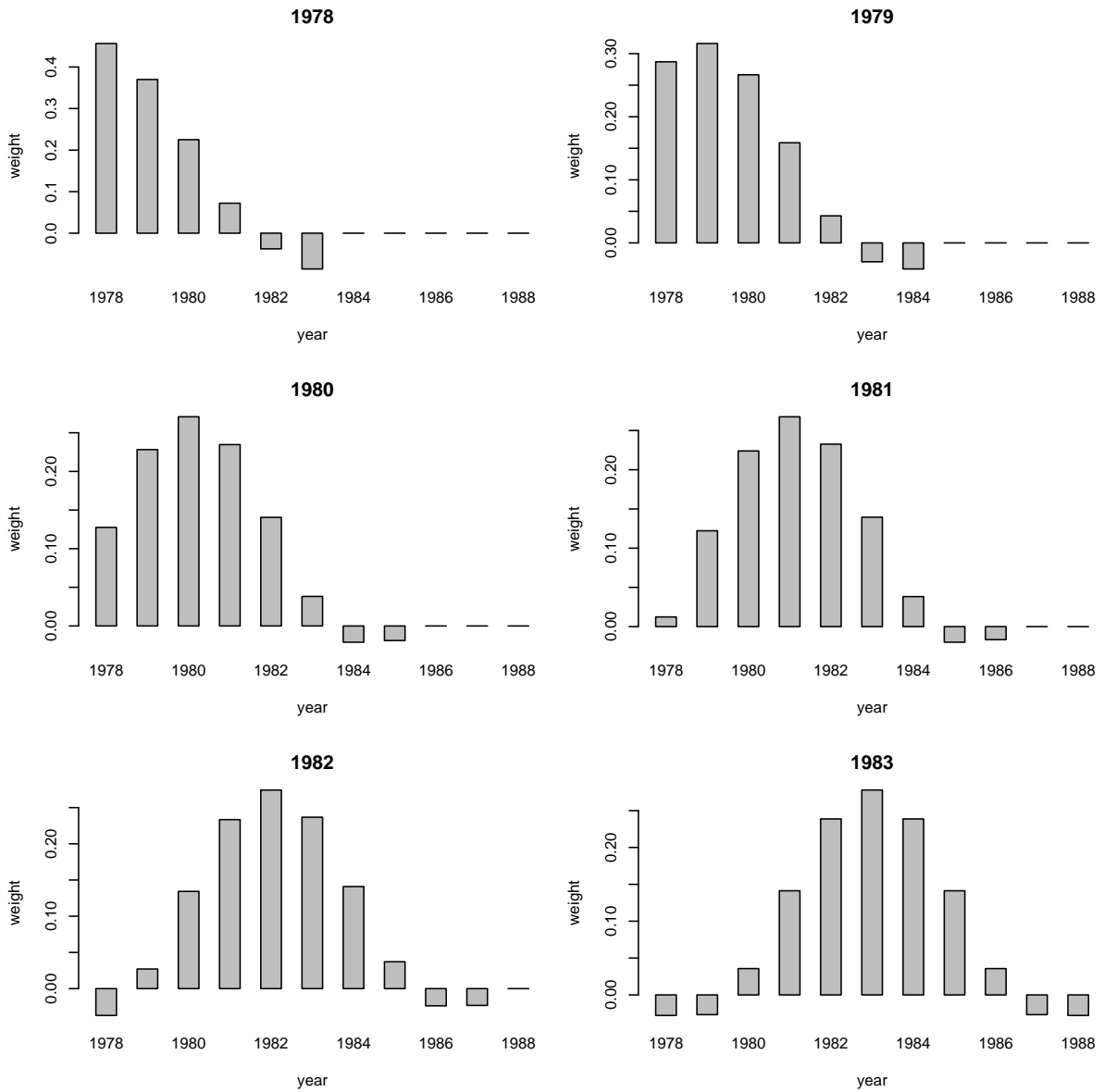
where the estimation of the cycle in this way is often referred to as linear detrending.

The use of such filters can have some practical limitations. This is because the estimate at certain points may depend on past or future values of the input series that have not been measured, or have not yet occurred. In such cases, one may estimate the missing observations to ensure the prediction of trend is the same length as the input series. In such circumstances, it is clear that the filtered series will be subject to revisions at end of the series, where forecasts are replaced with actual data. That is, for a symmetric filter of width  $2k + 1$ , filtered values  $k$  or more observations from the end of the series are subject to revision.

In the case of the Henderson filter, no forecasts are made, but the same consideration applies to the last  $k$  observations where an asymmetric, rather than symmetric, filter is applied. Figure D.1 shows the distribution of the asymmetric and symmetric weights for the Henderson filter of order 11.

Figure D.1

### Henderson Filter Weights



On the other hand, while the Hodrick-Prescott filter is indeed symmetric, this is only so at the centre observation, while observations sufficiently close to the centre are very near symmetric. Figure D.2 shows the distribution of weights for the Hodrick-Prescott filter with smoothing parameter  $\lambda = 6.25$  for a series 29 observations in length.

Both figures D.1 and D.2 suggest that at the third or fourth observation from the end-point, the filter is roughly symmetric, placing little weight on the missing observations. In practical terms, this would suggest that the estimated trend or cycle at a particular point would be subject to little revision, provided an additional two or three observations were available. Conversely, this also suggests that the last two or three observations of a series should be treated with caution.

Table D.1

**Revision History for Henderson and Hodrick-Prescott Filters**

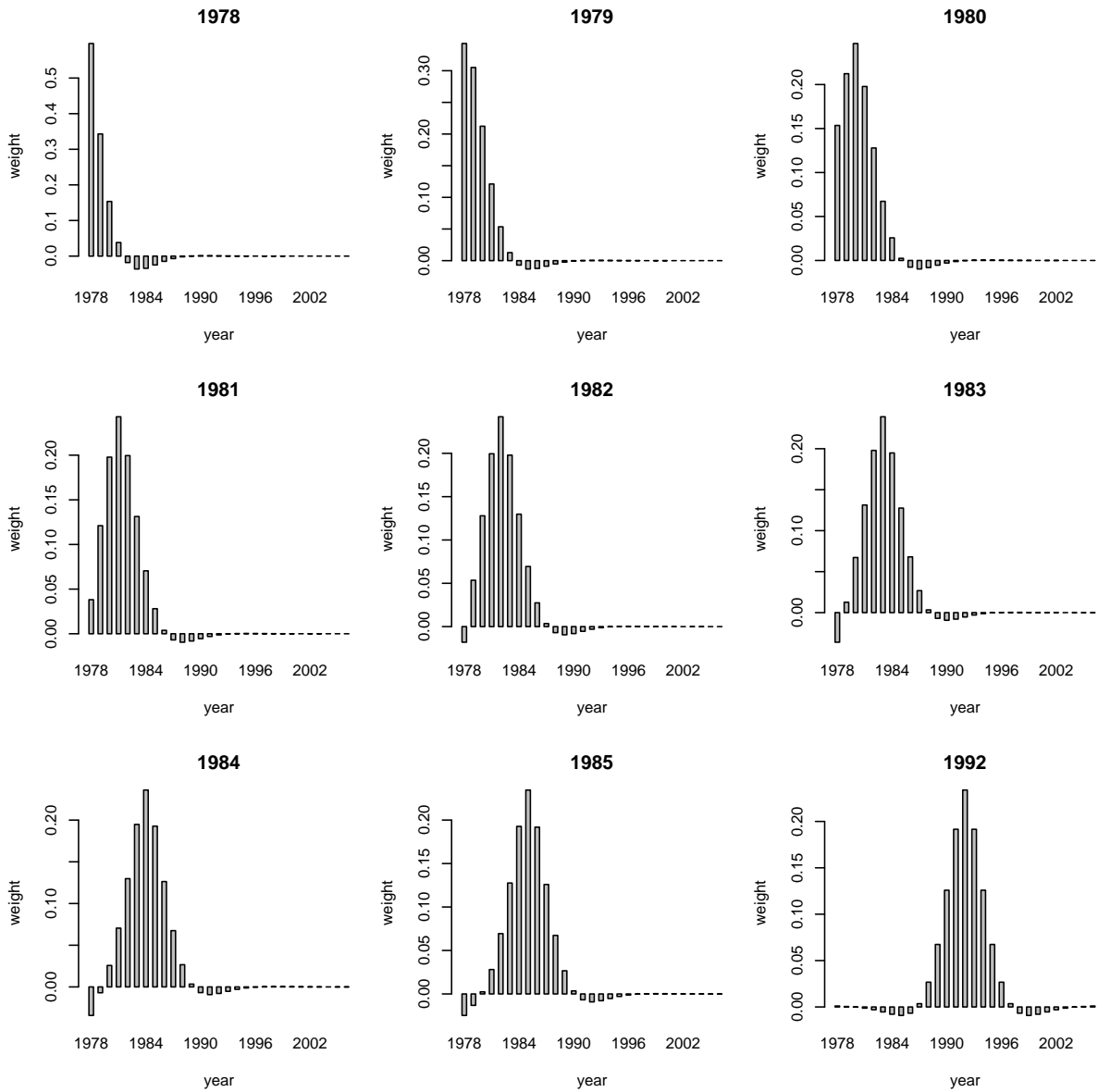
Year	Henderson filter					Hodrick-Prescott filter				
	number of periods from end-point									
1988	-2.72	0.53	0.70	-0.07	-0.19	-3.17	-0.20	1.06	1.30	1.07
1989	-4.50	-0.96	0.98	0.69	0.11	-4.08	-0.60	0.95	1.34	1.16
1990	-4.38	-1.27	0.87	1.12	0.40	-2.56	0.27	1.34	1.44	1.11
1991	-1.12	0.38	0.78	1.11	0.47	-0.59	1.68	2.17	1.82	1.21
1992	-0.59	2.35	2.19	0.96	0.44	-2.82	1.15	2.46	2.36	1.72
1993	-9.07	0.81	3.42	2.33	0.54	-11.06	-2.91	1.11	2.45	2.37
1994	-20.37	-5.01	2.33	3.61	1.64	-19.75	-8.45	-1.74	1.40	2.31
1995	-18.60	-4.82	1.81	2.97	1.55	-8.60	-4.38	-1.56	-0.03	0.59
1996	-13.44	-4.36	1.43	2.43	1.12	-0.25	-0.61	-0.80	-0.67	-0.45
1997	-8.81	-3.49	0.10	1.88	1.00	2.93	0.66	-0.43	-0.75	-0.70
1998	-3.95	-0.67	0.17	0.48	0.68	5.43	2.19	0.33	-0.51	-0.72
1999	-2.56	1.68	1.61	0.41	-0.32	5.61	2.88	1.05	0.04	-0.37
2000	-17.27	-5.12	-0.23	1.41	0.94	-11.69	-6.19	-2.40	-0.27	0.66
2001	-9.90	-2.57	-0.04	-0.19	0.21	1.78	-0.35	-1.12	-1.14	-0.86

Table D.1 shows the history of revisions for both filters as applied to the multifactor productivity series, for the years 1998 to 2001. The table suggests that, for the MFP series, the Henderson series has the larger revisions for the most recent endpoints, while the two filters perform similarly in periods that follow. However, while there are no revisions to observations more than five periods from the end-points for the Henderson filter, all observations will require revision for the Hodrick-Prescott filter. In a relative sense, however, the revisions are generally negligible for observations sufficiently removed from the end of the series.

At the end-points, the Henderson and Hodrick-Prescott filters do, in fact, contain a bias. This is largely because the end-points of the original series feature too prominently in the linear filter. One common way of dealing with this issue is, as in the case of missing data, by forecasting several observations ahead, and instead applying the filter to the extended series. However, this itself depends on the accuracy of the forecasts ex post. To this end, an ARIMA representation was sought for each time series

Figure D.2

### Hodrick-Prescott Filter Weights



discussed, but with limited success. For this reason, no estimation was used in determining the final trend and cycle components. If a large amount of emphasis was to be placed on the analysis of the trend component, this could be reconsidered.

## E Technical description of filters

### E.1 Hodrick-Prescott filter

The Hodrick-Prescott filter was introduced in a working paper in 1980, but first published in Hodrick and Prescott (1997). The filter was originally intended to help determine business cycles. The assumption is that a time series  $y_t$  is made up of an unobserved, non-stationary time trend,  $\tau_t$ , and a further (weakly) stationary cyclical component  $c_t$ . That is,

$$y_t = \tau_t + c_t, \quad t = 1, 2, \dots, T - 1, T \quad (2)$$

Hodrick and Prescott proposed a method of extracting  $\tau_t$  which requires the minimisation of the following cost

$$\text{Min}_{\{\tau_t\}_{t=1}^T} \left\{ \sum_{t=1}^T (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} [(\tau_{t+1} - \tau_t) - (\tau_t - \tau_{t-1})]^2 \right\}$$

The first term penalises the variance in the cyclical component, while  $\lambda$  describes how much the lack of smoothness in the trend contributes to the overall cost. The first order conditions can be written as

$$\hat{c} = \lambda \mathbf{F} \hat{\tau}$$

where

$$\mathbf{F} = \begin{pmatrix} \Delta^2 \mathbf{I}_T \end{pmatrix}' \begin{pmatrix} \Delta^2 \mathbf{I}_T \end{pmatrix} = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ -2 & 5 & -4 & 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & \dots & 0 \\ \vdots & & & & & & & \ddots & \vdots \\ 0 & \dots & & & & & 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & \dots & & & & & 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & \dots & & & & & 0 & 0 & 0 & 1 & -4 & 5 & -2 \\ 0 & \dots & & & & & 0 & 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix}$$

Substituting the first order conditions into (2) yields the solution

$$\hat{\tau} = (\lambda \mathbf{F}_{T \times T} + \mathbf{I}_T)^{-1} y$$

and, of course

$$\hat{c}_T = y_T - \hat{\tau}_T$$

Note that the Hodrick-Prescott filter is an approximate low-pass filter. This means that the filter permits low frequency components, and suppresses higher frequency components. That is, applying the filter results in an estimate of trend. Of course, subtracting the trend from the original series provides an estimate of the cycle, and in this case, the Hodrick-Prescott filter is referred to as an approximate high-pass filter. That is,

$$\hat{c} = \left[ \mathbf{I}_T - (\lambda \mathbf{F}_{T \times T} + \mathbf{I}_T)^{-1} \right] y$$

is an approximate high-pass filter.

The choice of the penalty parameter  $\lambda$  is somewhat contentious. However, commonly used values are 6.25 for annual data, 1600 for quarterly data and 129600 for monthly data. Figure E.1 shows the cycle component for MFP for several choices of smoothing parameter: 6.25, 8, 20 and 100. Similarly, figure E.2 shows the trend component for the same choices, including the limiting case. In general, the larger is  $\lambda$ , the smoother and straighter is the trend, and the larger in magnitude is the cycle. That is, as  $\lambda \rightarrow \infty$ , the trend approaches a smooth line. Except for the limiting case, each is a common choice for annual data. There is no appreciable difference between selected values of 6.25 and 8, and these choices yield the most reasonable results when compared with known economic events (see appendix A).

## E.2 Baxter-King filter

In the frequency domain, a time series is interpreted as a collection of periodic components. For example, the function  $\sin(2\omega)$  can be thought of as a periodic component with frequency  $\pi$ . In this context, we can think of a growth cycle as all the components with frequencies in a specified range for example,  $[\omega_l, \omega_h]$ . Then the cycle is a filtered series of the form

$$c_t = \sum_{-\infty}^{\infty} \beta_j y_{t-j}$$

Figure E.1

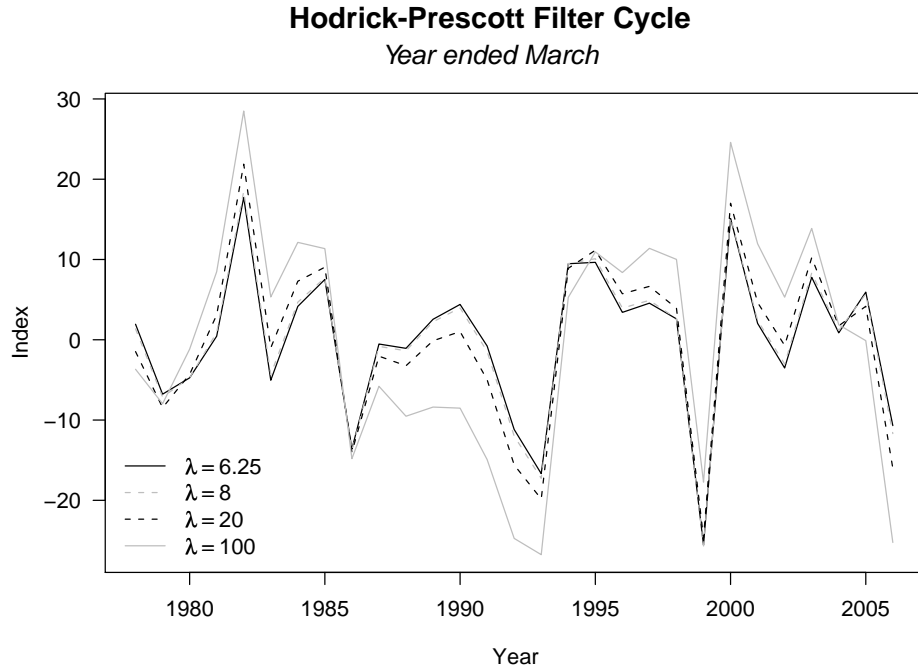
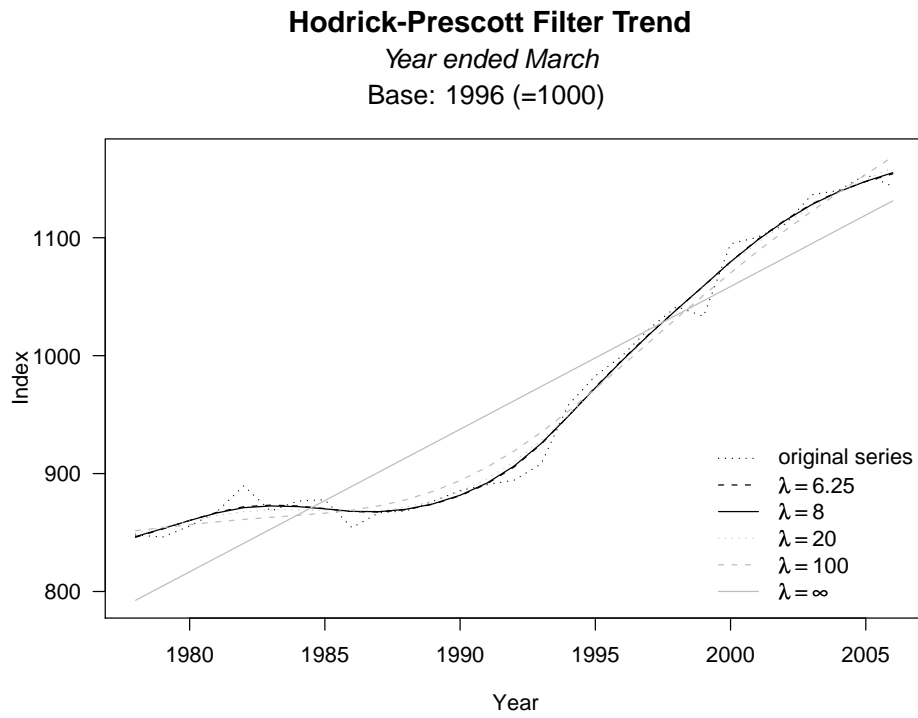


Figure E.2



where

$$\beta_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} \beta(e^{i\omega}) e^{-i\omega j} d\omega$$

and

$$\beta(e^{i\omega}) = \begin{cases} 1, & \omega_l \leq |\omega| \leq \omega_h \\ 0, & \text{otherwise} \end{cases}$$

The weights  $\beta_j$  can be shown to be

$$\beta_j = \begin{cases} \frac{\omega_l - \omega_h}{\sin(\frac{\pi}{j}) - \sin(\frac{\omega_h j}{j})}, & j = 0 \\ \frac{\sin(\omega_l j) - \sin(\omega_h j)}{\pi j}, & j \neq 0 \end{cases}$$

Outside the ideal band, the lower frequencies can be thought of as the trend, and the higher frequencies as the irregular component, or cycles that can be considered too short to be of interest. However, these components are not separated when applying the filter.

Of course, the ideal filter requires an infinite time series. In this case, an alternative finite approximation must be used:

$$\hat{c}_t = \sum_{j=-k}^k \alpha_j y_{t-j}$$

Baxter and King (1999) minimised:

$$Q = \int_{-\pi}^{\pi} \left| \sum_{j=-\infty}^{\infty} \beta_j e^{i\omega j} - \sum_{j=-k}^k \alpha_j e^{i\omega j} \right|^2 d\omega$$

subject to the additional constraint that  $\alpha(1) = 0$ . The solution can be shown to be:

$$\alpha_j = \beta_j + \phi, \quad j = -k, -k+1, \dots, 0, \dots, k-1, k$$

where

$$\phi = \frac{-\sum_{j=-k}^k \beta_j}{2k+1}$$

ensures the weights sum to 0.

The motivation for the Baxter-King filter was to extract US business cycles. For this purpose, Baxter and King (1999) suggest that the components of interest are from 1.5–2 years and 8 years in length. Table E.2 contains the suggested parameters. In addition,  $k$  is commonly chosen to be three times the frequency of the series. For quarterly data, this means  $k$  is set to 12, and for annual data, 3. 3 is the value chosen for the annual series examined in this paper.

Table E.3

**Suggested Parameters for Baxter-King Filter**

units	band width	$\omega_l$	$\omega_h$	$k$
months	1.5 to 8 years	$2\pi/18$	$2\pi/96$	12
quarters	1.5 to 8 years	$2\pi/6$	$2\pi/32$	12
years	2 to 8 years	$2\pi/2$	$2\pi/8$	3

In the context of productivity growth cycles, one needs to consider the appropriateness of different choices of interval for the band-pass. An interval of 2 to 8 years provides cycles most consistent with the other filters being considered. Figure E.3 shows cycles for the MFP data for different choices of interval. The results are broadly similar, except that shorter cycles have a tendency to be smoothed over longer cycles as the lower limit of the interval is increased.

### E.3 Henderson filter

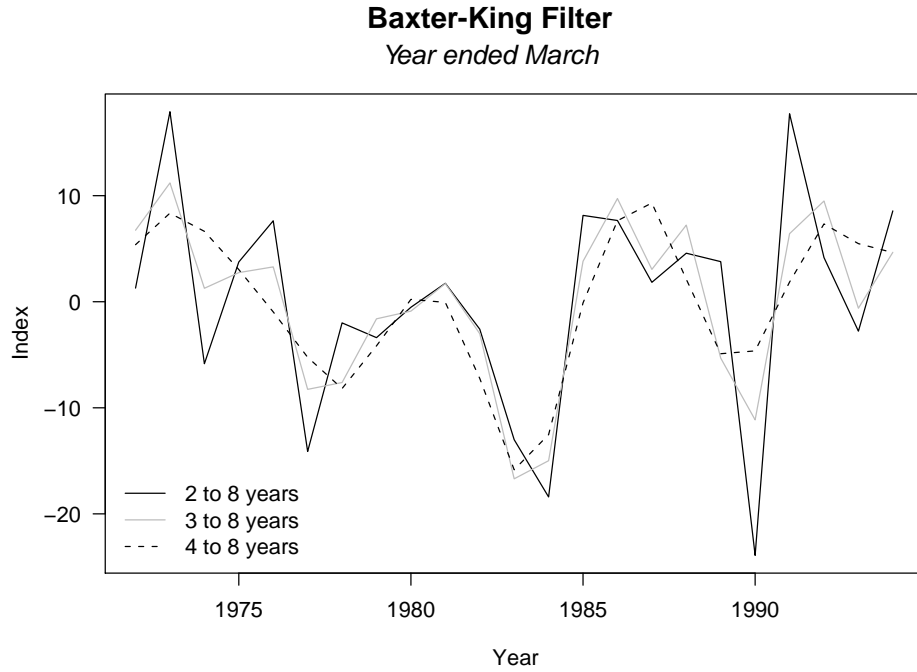
The Henderson filter is a special case of linear polynomial regression. We assume that there is a process,  $y_t$  such that

$$y_t = \tau(t) + c_t$$

We assume  $E(c_t) = 0$ , and that  $\tau(t)$  is a deterministic function in time such that  $E(y_t) = \tau(t)$ . If  $\tau(t)$  is differentiable, then it can be approximated locally by a polynomial,  $v(t)$ . That is,

$$\begin{aligned} \tau(t+j) &\approx v(t+j) \\ &= \sum_{k=0}^p \beta_k j^k \end{aligned}$$

Figure E.4



We assume that there are observations  $y_{t+h}$ ,  $j = 0, \pm 1, \pm 2, \dots, \pm h$ , and that  $p \leq 2h$ . Thus

$$y_t = \sum_{k=0}^p \beta_k j^k + c_{t+j}$$

In matrix notation the relation can be expressed

$$\begin{bmatrix} y_{t-h} \\ \vdots \\ y_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & -h & \cdots & (-h)^p \\ 1 & -(h-1) & \cdots & [-(h-1)]^p \\ \vdots & \vdots & \cdots & \vdots \\ 1 & 0 & \cdots & \vdots \\ 1 & h-1 & \cdots & (h-1)^p \\ 1 & h & \cdots & h^p \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_p \end{bmatrix} + \begin{bmatrix} c_{t-h} \\ \vdots \\ c_{t+h} \end{bmatrix}$$

or, alternatively

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{c}$$

Defining

$$\mathbf{U} = \begin{bmatrix} u_h & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & u_{h-1} & \cdots & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\ 0 & 0 & \cdots & u_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & u_0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & u_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & u_{h-1} & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 \cdots & 0 & u_h \end{bmatrix}$$

the weighted least squares (WLS) estimate of  $\beta$  is

$$\hat{\beta} = (\mathbf{X}'\mathbf{U}\mathbf{X})^{-1} \mathbf{X}'\mathbf{U}\mathbf{y}$$

Thus,

$$\begin{aligned} \hat{\tau}(t) &= \hat{\beta}_0 \\ &= \mathbf{e}'_1 (\mathbf{X}'\mathbf{U}\mathbf{X})^{-1} \mathbf{X}'\mathbf{U}\mathbf{y} \\ &= \mathbf{w}'\mathbf{y} \\ &= \sum_{j=-h}^h w_j y_{t-j} \end{aligned}$$

or

$$\mathbf{w} = \mathbf{e}'_1 (\mathbf{X}'\mathbf{U}\mathbf{X})^{-1} \mathbf{X}'\mathbf{U}$$

The Henderson filter arises where  $p = 3$ , and the weights are chosen in such a way that  $\text{Var}[\Delta^3 \hat{g}(t)]$  is minimised, subject to the constraints

$$\sum_{j=-h}^h w_j = 1 \text{ and } \sum_{j=-h}^h w_j j^k = 0, \quad k = 1, 2, 3$$

It can be show that the solution is

$$w_j = \frac{315}{8} \frac{[(h+1)^2 - j^2][(h+2)^2 - j^2][(h+3)^2 - j^2][3(h+2)^2 - 11j^2 - 16]}{(h+2)[(h+2)^2 - 1][4(h+2)^2 - 9][4(h+2)^2 - 25]}$$

where  $j = 0, \pm 1, \dots, \pm h$ .

In the case of the Henderson filter, a well known set of asymmetric weights exist that can be used at the end-points where there are missing observations. This is known as the Musgrave method, and the weights are collectively referred to as a surrogate Henderson filter. These are defined in Doherty (2001) as

$$u_{m,r} = w_{-h+(r-1)} + \frac{1}{m} \sum_{j=-h+m}^h w_j + \frac{\left(r - \frac{m+1}{2}\right) \frac{4/\pi}{(\bar{I}/\bar{C})^2}}{1 + \frac{m(m-1)(m+1)}{12} \frac{4/\pi}{(\bar{I}/\bar{C})^2}} \sum_{j=-h+m}^h \left[ (j+h+1) - \frac{m+1}{2} \right] w_j$$

where  $r = 1, 2, \dots, m$  and  $m = h+1, \dots, 2h$ , and  $\bar{I}/\bar{C}$  is the noise to trend ratio. A surrogate containing  $m$  terms is applied to points in a series where the number of missing observations on either side is  $2h+1-m$ . To illustrate, the 11-term Henderson filter has 5 sets of surrogate weights, with lengths ranging from 6 to 10. The surrogate of length 7, say, applies at points in the series where there are 4 missing observations. That is, the second and second-to-last observations:

$$\hat{\tau}(2) = u_{7,7}y_1 + u_{7,6}y_2 + \dots + u_{7,2}y_6 + u_{7,1}y_7$$

and

$$\hat{\tau}(T-1) = u_{7,1}y_{T-6} + u_{7,2}y_{T-5} + \dots + u_{7,6}y_{T-1} + u_{7,7}y_T$$

While the Henderson filter is a common method of smoothing data or estimating trend, it is probably not commonly used to estimate business or growth cycles. Regardless, it is typical to fix the order of the filter at 23 for quarterly data, and 11 for annual data. In addition, in determining the surrogate weights, a noise to trend ratio of 4.5 is typical for quarterly data, and 3.5 for annual data.

## References

- [1] Baxter, M & King, R, (1999). "Measuring business cycles; Approximate bandpass filters for economic time series." *Review of Economics and Statistics*, 81, p. 575–593.
- [2] Dalziel, P & Lattimore, R, (2001). *The New Zealand macroeconomy: A briefing on the reforms and their legacy*. Oxford University Press.
- [3] Doherty, M, (2001). "The surrogate Henderson Filters in X-11." *Australian & New Zealand Journal of Statistics*, Volume 43, No. 4, December 2001.
- [4] Haywood, E, (1972). "The dating of post war business cycles in New Zealand 1946–1970." Reserve Bank of New Zealand, Research paper No. 4, March.
- [5] Haywood, E & Campbell, C, (1976). The New Zealand Economy: measurement of economic fluctuations and indicators of economic activity. *Reserve Bank of New Zealand, Research paper No. 19, May*.
- [6] Henderson, R, (1916). "Note on Graduation by Adjusted Average." *Transactions of the American Society of Actuaries*, 17, p 43–48.
- [7] Hodrick, R J & Prescott, E C, (1997). "Postwar U.S. business cycles; An empirical investigation." *Empirical Economics*, 19, p 493–500.
- [8] Kenny, P B & Durbin, J, (1982). "Local trend estimation and seasonal adjustment of economic and social time series." *Journal of the Royal Statistical Society, Series A*, 145, p 1–41.
- [9] Kim, K, Buckle, R A & Hall, V B, (1994). "Dating New Zealand Business Cycles." *The Graduate School of Business and Government Management Working Paper Series 6/94*, Victoria University of Wellington, New Zealand.
- [10] Reserve Bank of New Zealand, (2007). *New Zealand economic and financial chronology 1993–2005*. Available at: <http://www.rbnz.govt.nz/about/whatwedo/1613129.html> [accessed 20 August 2007].
- [11] Hall, VB, McDermott, CJ , (2007). "The New Zealand Business Cycle." Accepted for publication in the Bergstrom Memorial and Dedication Issue of *Econometric Theory*.
- [12] Zhang, X, Conn, L, (2007). "Estimating the Cyclical Component from Annual Time Series." *Australian Bureau of Statistics, Research paper*.